CHAPTER 20 -- A. C. CIRCUITS

20.1) 12 volts peak to peak implies that the amplitude of the voltage is 6 volts ($V_o = V_{pp}/2$).

a.) The voltage as a function of time has the general form:

$$V(t) = V_o \sin (2\pi\nu t).$$

For this problem, V_o = 6 volts and ν = 2500 hertz. Substituting in, we get:

$$V(t) = 6 \sin (15{,}708\ t).$$

b.) For the RMS voltage:

$$\begin{aligned} V_{RMS} &= .707V_o \\ &= .707(6 \text{ volts}) \\ &= 4.24 \text{ volts.} \end{aligned}$$

c.) AC ammeters read RMS values. That means i_{RMS} = 1.2 amps. The maximum current is the amplitude (i_o), therefore:

$$\begin{aligned} i_{RMS} &= .707 i_o \\ \Rightarrow \quad i_o &= i_{RMS}/.707 \\ &= (1.2 \text{ A})/.707 \\ &= 1.7 \text{ amps.} \end{aligned}$$

Note: You have to be careful when dealing with AC circuits. A device that can only withstand 1.5 amps maximum is not going to be safe if put in an AC circuit whose RMS current is 1.2 amps. Why? Because there will be periods in the current's cycle during which the current's value will exceed 1.5 amps (it'll go all the way up to 1.7 amps according to our calculations above). Obviously the problem will not arise if the device is rated at 1.5 amps RMS, which is the way most devices are rated (that is, in terms of their RMS currents). Nevertheless, you need to be sure you know what numbers you are dealing with when working with AC circuits in the everyday world.

20.2) The trick to unscrambling questions like these is to remember: 1.) the inductive reactance X_L measures the amount of <u>resistance</u> to current flow there is in an AC circuit due to the presence of the inductor--when X_L is LARGE, current will be SMALL, and vice versa (the same can be said of the capacitive reactance X_C); and 2.) the amount of current in any circuit is mirrored by the size of the voltage drop across the circuit's resistor--when the resistor's voltage is HIGH, the current must be HIGH, and vice versa. With all of this in mind:

a.) False: The time-average voltage (V_{avg}) across a resistor in an AC circuit is zero. The RMS voltage across a resistor in an AC circuit is equivalent to the constant DC voltage that will provide the same amount of power to the resistor as does the alternating voltage. In no case will these two be the same.

b.) True: The AC version of Ohm's Law states that $V_{RMS} = i_{RMS}R$.

c.) False: The capacitive reactance (X_C) measures the capacitor's resistance to charge flow in the circuit. If that value is large, there will be a small current in the circuit. A small current implies a small voltage across the resistor.

d.) True: Again, capacitive reactance measures the resistive nature of the capacitor in an AC circuit. When it is large, the current in the circuit will be small; when it is small (as is the case here), the current in the circuit will be large.

e.) True: The capacitive reactance is equal to $1/(2\pi\nu C)$. Decreasing the frequency increases the resistive nature of the circuit. This decreases the current. A decrease in current decreases the voltage across the resistor. As the sum of the voltage differences of the circuit elements must equal the power supply voltage at a given instant, decreasing the voltage across the resistor must elicit an equal increase in voltage across the capacitor.

f.) True: The capacitive reactance (X_c) numerically equals $1/(2\pi\nu C)$. Increasing the capacitance for a given frequency decreases X_c. If the resistive nature of the capacitor (X_c) decreases, the current will go up.

g.) False: Decreasing the frequency increases the resistive nature of the capacitor ($X_C = 1/(2\pi\nu C)$ goes up when ν goes down). This drops the current, which drops the voltage across the resistor.

20.3)

a.) True: This is still true (see Problem 20.2b above).

b.) False: The inductive reactance (X_L) measures the resistive nature of the inductor. If that value is large, there will be a small current in the circuit. A small current implies a small voltage across the resistor.

c.) True: Again, if a measure of the resistive nature of the inductor (i.e., X_L) is small, the current in the circuit will be large.

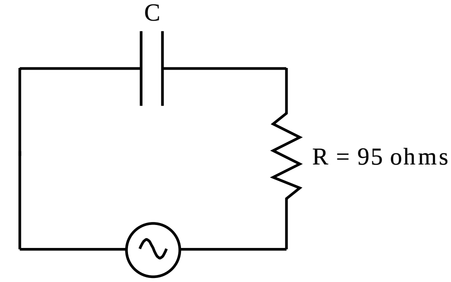
d.) False: The inductive reactance is equal to $2\pi\nu L$. Decreasing the frequency decreases the resistive nature of the circuit, increasing the current. An increase in current increases the voltage across the resistor, which means the voltage across the inductor must decrease (this is similar to the reasoning presented in Problem 20.2e).

e.) False: Increasing the inductance increases X_L. This will decrease the current in the circuit.

f.) True: Decreasing the frequency will decrease X_L. This will increase the current in the circuit which, in turn, will increase the voltage across the resistor.

20.4) Capacitors allow high frequency signals to pass; inductors stop high frequency. The resistor that dissipates the most power is the one with the most current in it. That will be the resistor in series with the capacitor, or the 12 Ω resistor.

20.5) The circuit is shown to the right.



a.) The capacitive reactance X_c is a part of the impedance (note that $X_L = 0$). Knowing the expression for impedance, we write:

$$Z = [R_{net}{}^2 + (X_L - X_c)^2]^{1/2}$$
$$= [R_{net}{}^2 + (-X_c)^2]^{1/2}$$
$$\Rightarrow \quad X_c = [Z^2 - R_{net}{}^2]^{1/2}$$
$$= [(220\ \Omega)^2 - (95\ \Omega)^2]^{1/2}$$
$$= 198.4\ \Omega.$$

b.) We need to determine the capacitance C. We know the capacitive reactance is $X_c = 1/(2\pi\nu C)$, and we know that when the frequency is 300 hertz, X_c equals 198.4 Ω (from above). Manipulating the X_c expression, then plugging in, we get:

$$\begin{aligned} C &= 1/(2\pi\nu X_c) \\ &= 1/[2\pi(300 \text{ hz})(198.4\ \Omega)] \\ &= 2.67\text{x}10^{-6} \text{ farads.} \qquad \text{(This is } 2.67\ \mu\text{f).} \end{aligned}$$

Knowing C, we can determine Z at 1000 hz:

$$\begin{aligned} Z &= [R_{net}{}^2 + (X_L - X_c)^2]^{1/2} \\ &= [R_{net}{}^2 + [(0) - (1/(2\pi\nu C))^2]^{1/2} \\ &= [(95\ \Omega)^2 + [(0) - 1/(2\pi(1000 \text{ hz})(2.67\text{x}10^{-6} \text{ f}))]^2]^{1/2} \\ &= 112.2\ \Omega. \end{aligned}$$

c.) We know V_{max} = 80 volts.

$$\begin{aligned} V_{RMS} &= .707V_{max} \\ &= .707(80 \text{ v}) \\ &= 56.56 \text{ volts.} \end{aligned}$$

From Ohm's Law:

$$\begin{aligned} V_{RMS} &= i_{RMS}(Z) \\ \Rightarrow \quad i_{RMS} &= V_{RMS}/(Z) \\ &= (56.56 \text{ v})/(112.2\ \Omega) \\ &= .504 \text{ amps.} \end{aligned}$$

20.6) An RL circuit:

a.) The inductive reactance X_L is a part of the impedance (note that $X_C = 0$). We also know that the net resistance is $R_{net} = R + r_L$, where r_L is the resistor-type resistance involved in the wire making up the inductor's coils. This value will be $R_{net} = 12\ \Omega + 8\ \Omega = 20\ \Omega$. Knowing the impedance Z, we can write:

$$Z = [R_{net}{}^2 + (X_L - X_c)^2]^{1/2}$$
$$= [R_{net}{}^2 + X_L{}^2]^{1/2}$$
$$\Rightarrow \quad X_L = [Z^2 - R_{net}{}^2]^{1/2}$$
$$= [(60\ \Omega)^2 - (20\ \Omega)^2]^{1/2}$$
$$= 56.57\ \Omega.$$

b.) We need to determine the inductance L. We know the inductive reactance is $X_L = 2\pi\nu L$, and we know that when the frequency is 240 hertz, X_L equals 56.57 Ω (from above). Manipulating the X_c expression, then plugging in, we get:

$$L = X_L/2\pi\nu$$
$$= (56.57\ \Omega)/[2\pi(240\text{ hz})]$$
$$= 3.75\text{x}10^{-2}\text{ henrys} \qquad \text{(this is 37 mH).}$$

Knowing L, we can determine Z at 1000 hz using:

$$Z = [R_{net}{}^2 + (X_L - X_c)^2]^{1/2}$$
$$= [R_{net}{}^2 + [(2\pi\nu L) - (0)]^2]^{1/2}$$
$$= [(20\ \Omega)^2 + [2\pi(1000\text{ hz})(3.7\text{x}10^{-2}\text{ H})]^2]^{1/2}$$
$$= 233.3\ \Omega.$$

c.) We know V_{RMS} = 70 volts. Using Ohm's Law:

$$V_{RMS} = i_{RMS}(Z)$$
$$\Rightarrow \quad i_{RMS} = V_{RMS}/(Z)$$
$$= (70\text{ v})/(233.3\ \Omega)$$
$$= .3\text{ amps.}$$

20.7) $V(t) = 140 \sin(1100 t)$, $R = 12\ \Omega$, $L = 60\text{x}10^{-3}$ henrys, and $C = 12\text{x}10^{-6}$ farads.

a.) The angular frequency is $\omega = 2\pi\nu$, which means the frequency ν is embedded in the sine function's argument. We know that $\omega = 1100$ radians per second, so we can write:

$$2\pi\nu = 1100$$

$$\Rightarrow \quad \nu = 175 \text{ hz.}$$

b.) Capacitive reactance:

$$\begin{aligned} X_c &= 1/(2\pi\nu C) \\ &= 1/[2\pi(175 \text{ hz})(12\text{x}10^{-6} \text{ f})] \\ &= 75.79\ \Omega. \end{aligned}$$

Note: Technically, it is usually best to use variables that are given in a problem, versus variables that have been derived in a previous section. I have used the frequency-explicit expression for capacitive reactance (even though the frequency had to be calculated in Part a) because that is the expression you will most often use when trying to determine such quantities. My apologies for the apparent inconsistency.

c.) Inductive reactance:

$$\begin{aligned} X_L &= 2\pi\nu L \\ &= 2\pi(175 \text{ hz})(60\text{x}10^{-3} \text{ H}) \\ &= 65.97\ \Omega. \end{aligned}$$

d.) Impedance:

$$\begin{aligned} Z &= [R_{net}{}^2 + (X_L - X_c)^2]^{1/2} \\ &= [(12\ \Omega)^2 + [(65.97\ \Omega) - (75.79\ \Omega)]^2]^{1/2} \\ &= 15.5\ \Omega. \end{aligned}$$

e.) Phase shift:

$$\begin{aligned} \phi &= \tan^{-1} [(X_L - X_c)/R_{net}] \\ &= \tan^{-1} [(65.97 - 75.79)/(12)] \\ &= -39.29^o. \end{aligned}$$

The negative sign means the voltage lags the current.

f.) RMS voltage of the power supply:

$$\begin{aligned} V_{RMS} &= .707V_o \\ &= .707(140 \text{ volts}) \end{aligned}$$

$= 98.98$ volts.

g.) RMS current in the circuit: Using Ohm's Law:

$$i_{RMS} = V_{RMS}/Z$$
$$= (98.98 \text{ v})/(15.5\ \Omega)$$
$$= 6.39 \text{ amps.}$$

h.) Resonance frequency:

$$\nu_{res} = [1/(2\pi)]\ [1/LC]^{1/2}$$
$$= [1/(2\pi)]\ [1/[(60\text{x}10^{-3})(12\text{x}10^{-6})]]^{1/2}$$
$$= 187.57 \text{ Hz.}$$

i.) At resonance, the frequency is 187.57 hertz. The time-dependent voltage at that frequency is:

$$V(t) = 140 \sin (2\pi\nu_{res}\ t)$$
$$= 140 \sin [2\pi(187.57)\ t]$$
$$= 140 \sin (1178.5\ t).$$

j.) Impedance at resonance: At resonance, $X_L - X_c = 0$, so:

$$Z = [R_{net}{}^2 + (0)^2]^{1/2}$$
$$= R_{net}$$
$$= 12\ \Omega.$$

Note: This is the SMALLEST impedance the circuit will ever experience.

k.) RMS current at resonance:

$$i_{RMS} = V_{RMS}/Z$$
$$= (98.98 \text{ v})/(12\ \Omega)$$
$$= 8.25 \text{ amps.}$$

Note that the LARGEST current the circuit will ever experience will be $i_o = i_{RMS}/.707 = 11.7$ amps.

20.8)

a.) Impedance matching requires the use of a transformer the turns-ratio of which must be:

$$N_p/N_s = (Z_{st}/Z_{load})^{1/2}.$$

Using the information given in the problem:

$$N_p / N_s = [Z_{st} / Z_{load}]^{1/2}$$
$$(200)/(N_s) = [(237\Omega)/(12\ \Omega)]^{1/2}$$
$$\Rightarrow \quad N_s = (200)/(4.444)$$
$$= 45.$$

A 200 winds primary with a 45 winds secondary will make the signal from the 237 Ω impedance stereo think it is entering a 237 Ω impedance speaker.

b.) As the number of winds in the secondary are less than in the primary, we are looking at a step-down transformer.